Duality Constrained Parameterization of Vector and Axial Nucleon Form Factors

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We present new parameterizations of vector and axial nucleon form factors. We maintain an excellent descriptions of the form factors at low momentum transfers, where the spatial structure of the nucleon is important, and use the Nachtman scaling variable ξ to relate elastic and inelastic form factors and impose quark-hadron duality constraints at high momentum transfers where the quark structure dominates. We use the new vector form factors to re-extract updated values of the axial form factor from neutrino experiments on deuterium. We obtain an updated world average value from neutrino and pion electroproduction experiments of $M_A = 1.0155 \pm 0.0136\ GeV/c^2$. Our parameterizations are useful in modeling neutrino interactions at low energies (e.g. for neutrino oscillations experiments). The predictions for high momentum transfers can be tested in the next generation electron and neutrino scattering experiments.

The nucleon vector and axial elastic form factors have been measured for more than 50 years in e^-N and νN scattering. At low Q^2, a reasonable description of the proton and neutron elastic form factors is given by the dipole approximation The dipole approximation is a lowest-order attempt to incorporate the non-zero size of the proton into the form factors. The approximation assumes that the proton has a simple exponential spatial charge distribution, $\rho(r) = \rho_0 e^{-r/r_0}$, where r_0 is the scale of the proton radius. Since the form factors are related in the non-relativistic limit to the Fourier transform of the charge and magnetic moment distribution, the above $\rho(r)$ yields the dipole form defined by:

$$G_D^{V,A}(Q^2) = \frac{C^{V,A}}{\left(1 + \frac{Q^2}{M_{V,A}^2}\right)^2}.$$

Here $C^{V,A}$= (1,g_A), g_A = -1.267, M_V^2 = 0.71 $(GeV/c)^2$, and M_A = 1.015 GeV/c (as discussed below).

Since M_A is not equal to M_V, the distribution of electric and axial charge are different. However, the magnetic moment distribution were assumed to have the same spatial dependence as the charge distribution (*i.e.*, form factor scaling). Recent measurements from Jefferson Lab have shown an unexpected structure in the ratio of $\frac{\mu_p G_{Ep}}{G_{Mp}}$ at high Q^2 challenging the validity of form factor scaling [1–3] and resulting in new updated parameterizations of the form factors (See [6] and [10] and references therein). In this paper we present parameterizations that simultaneously satisfy constraints at low Q^2 where the spatial structure of the nucleon is important, as well as at high Q^2 where the quark structure is important. A violation of form-factor scaling is expected within the framework of quark-hadron duality [7]. We then use our new vector form factors to re-extract updated values of the axial form factor from a re-analysis of previous neutrino scattering data on deuterium and present a new parameterization for the axial form factor within the framework of quark-hadron duality.

The new parameterizations presented in this paper are referred to as the duality based "BBBA07" parameterization. Our updated parameterizations feature the following: (1) Improved functional form that uses Nachtman scaling variable ξ to relate elastic and inelastic vector and axial nucleon form factors; (2) Yield the same values as Arrington and Sick [9] for $Q^2 < 0.64(GeV/c)^2$, while satisfying quark-hadron duality constraints at high-Q^2.

For vector form factors our fit functions are $A_N(\xi)$ (i.e. $A_{Ep}(\xi)$, $A_{Mp}(\xi)$, $A_{En}(\xi)$, $A_{Mn}(\xi)$) multiplied an updated Kelly[10] type parameterization of one of the proton form factors. The Kelly parameterization is:

$$G^{Kelly}(Q^2) = \frac{\sum_{k=0}^{m} a_k \tau^k}{1 + \sum_{k=1}^{m+2} b_k \tau^k},$$

where $a_0 = 1$, $m = 1$, and $\tau = Q^2/4M_N^2$. (M_N is proton, neutron, or average nucleon mass for proton, neutron, and axial form factors, respectively). The datasets used by Kelly[10] to fit G_{Ep} and G_{Mp}/μ_p ($\mu_p = 2.7928$, $\mu_n = -1.913$) are described in [10]. Our parameterization employs the as-published Kelly parameterization to G_{Ep}^{Kelly} and an updated set of parameters for $G_{MP}^{Kelly}(Q^2)$ that includes the recent BLAST[8] results. The parameters used for G_{Ep}^{Kelly} and G_{Mp}^{Kelly} are listed in Table I, and $A_N(\xi)$ is given by

$$A_N(\xi) = \sum_{j=1}^{n} P_j(\xi)$$
$$P_j(\xi) = p_j \prod_{k=1,k\neq j}^{n} \frac{\xi - \xi_k}{\xi_j - \xi_k}.$$

Each P_j is a LaGrange polynomial in the Nachtman variable, $\xi = \frac{2}{(1+\sqrt{1+1/\tau})}$. The ξ_j are equidistant "nodes" on an interval $[0, 1]$ and p_j are the fit parameters that have an additional property $A_N(\xi_j) = p_j$. The functional form $A_N(\xi)$ (for G_{Ep}, G_{Mp}, G_{En}, G_{Mn}) is used with seven p_j parameters at ξ_j=0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1.0. In the fitting procedure described below, the parameters of $A_N(\xi)$ are constrained to give the same vector form factors as the recent low Q^2 fit of Arrington and Sick [9] for

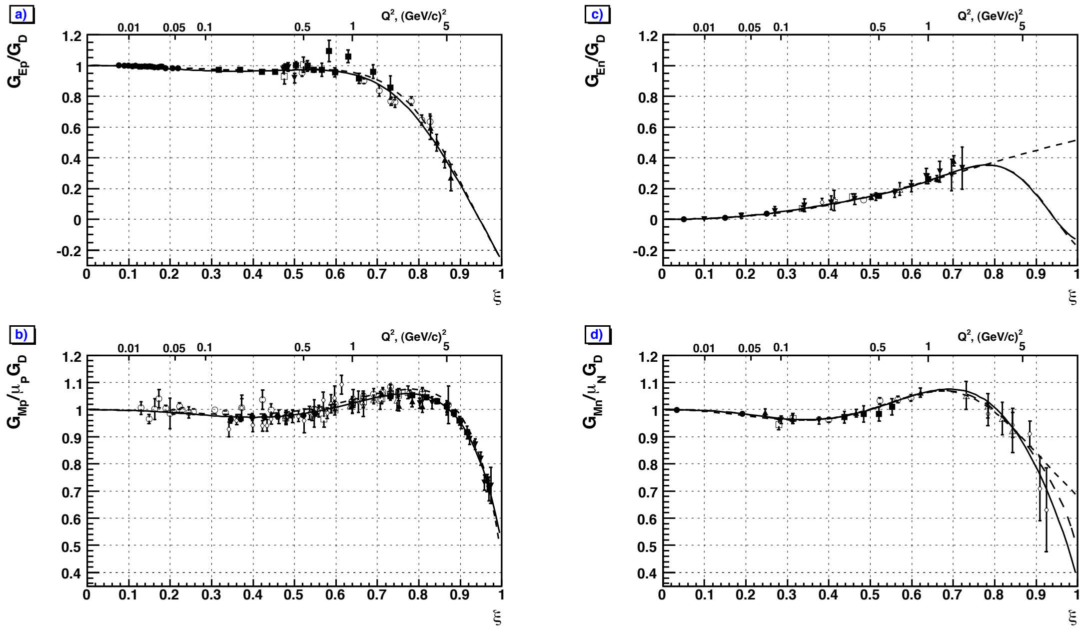

FIG. 1: Ratios of G_{Ep} (a), G_{Mp}/μ_p (b), G_{En} (c) and G_{Mn}/μ_n (d) to G_D. The short-dashed line in each plot is the old Kelly parameterizations (old Galster for G_{En}). The solid line is our new BBBA07 parameterization for $\frac{d}{u} = 0.0$, and the long-dashed line is BBBA07 for $\frac{d}{u} = 0.2$. The values of ξ and the corresponding values of Q^2 are shown on the bottom and top axis.

$Q^2 < 0.64(GeV/c)^2$ (as that analysis includes coulombs corrections which modify G_{Ep},two photon exchange corrections which modify G_{Mp} and G_{Mn}). Since the published form factor data do not have these corrections, this constraint is implemented by including additional $''fake''$ data points for $Q^2 < 0.64(GeV/c)^2$.

Our fits to the form factors are:

$$\begin{aligned} G_{Mp}(Q^2)/\mu_p &= A_{Mp}(\xi) \times G_{Mp}^{Kelly}(Q^2) \\ G_{Ep}(Q^2) &= A_{Ep}(\xi) \times G_{Ep}^{Kelly}(Q^2) \\ G_{Mn}(Q^2)/\mu_n &= A_{Mn}^{a,b}(\xi) \times G_{Mp}(Q^2)/\mu_p \\ G_{En}(Q^2) &= A_{En}^{a,b}(\xi) \times G_{Ep}(Q^2) \times \left(\frac{a\tau}{1+b\tau}\right), \end{aligned}$$

where we use our updated parameters in the Kelly parameterizations. For G_{En} the parameters a=1.7 and b=3.3 are the same as in the Galster[4] parametrization and ensure that dG_{En}/dQ^2 at for $Q^2 = 0$ is in agreement with measurements. For convenience, we also provide fits for the form factors G_{Ep} and G_{Mp}/μ_p that give very close to the same values, but use the dipole form instead:

$$\begin{aligned} G_{Ep}(Q^2) &= A_{Ep-dipole}(\xi) \times G_D^V(Q^2) \\ G_{Mp}(Q^2)/\mu_p &= A_{Mp-dipole}(\xi) \times G_D^V(Q^2) \end{aligned}$$

The values $A(\xi)$=p_1 at ξ_1=0 ($Q^2 = 0$) for G_{Mp}, G_{Ep}, G_{En}, G_{Mn} are set to to 1.0. The value $A(\xi)$=p_7 at ξ_j=1 ($Q^2 \to \infty$) for G_{Mp} and G_{Ep} is set to 1.0. The value $A(\xi)$=p_j at ξ_j=1 for G_{Mn} and G_{En} are fixed by con-

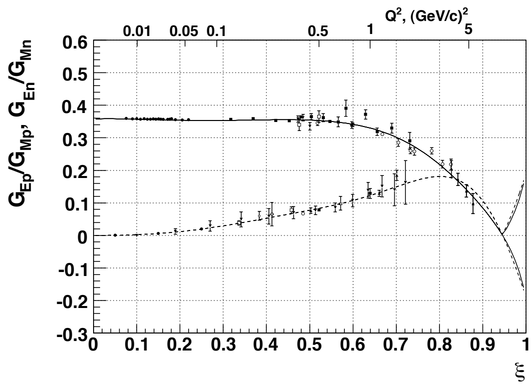

FIG. 2: The constraint used in fitting G_{En} stipulates that $G^2_{En}/G^2_{Mn} = G^2_{Ep}/G^2_{Mp}$ at high ξ. The solid line is $\frac{G_{Ep}}{|G_{Mp}|}$ and $\frac{|G_{Ep}|}{|G_{Mp}|}$, and the short-dashed line is $\frac{G_{En}}{|G_{Mn}|}$ and $\frac{|G_{En}|}{|G_{Mn}|}$.

straints from quark-hadron duality. Quark-hadron duality implies that the ratio of neutron and proton magnetic form factors should be the same as the ratio of the corresponding inelastic structure functions $\frac{F_{2n}}{F_{2p}}$ in the ξ=1 limit. (Here $F_2 = \xi \sum_i e_i^2 q_i\,(\xi)$)

$$\frac{G^2_{Mn}}{G^2_{Mp}} = \frac{F_{2n}}{F_{2p}} = \frac{1+4\frac{d}{u}}{4+\frac{d}{u}} = \left(\frac{\mu_n^2}{\mu_p^2}\right) A^2_{Mn}(\xi=1)$$

We ran fits with two different values of $\frac{d}{u}$ at the ξ=1 limit: $\frac{d}{u}$ =0 and 0.2 (corresponding to $\frac{F_{2n}}{F_{2p}}$ = 0.25 and

	a_1	b_1	b_2	b_3	χ^2/ndf
G_{Ep}^{Kelly}	-0.24	10.98	12.82	21.97	0.78
G_{Mp}^{Kelly}	0.17195	11.2595	19.3219	8.33346	1.03

TABLE I: Parameters for G_{Ep}^{Kelly} and G_{Mp}^{Kelly}. Our parameterization employs the as-published Kelly parameterization to G_{Ep}^{Kelly} and an updated set of parameters for $G_{MP}^{Kelly}(Q^2)$ that includes the recent BLAST[8] results.

ξ, Q^2	p_1 0, 0	p_2 0.167, 0.029	p_3 0.333, 0.147	p_4 0.500, 0.440	p_5 0.667, 1.174,	p_6 0.833, 3.668	p_7 1.0, ∞
A_{Ep}	1.	0.992707	0.989825	0.997507	0.981319	0.934137	1.
A_{Mp}	1.	1.001060	0.999111	0.997339	1.000996	1.000214	1.
$A_{Ep-dipole}$	1.	0.983874	0.963178	0.974797	0.913645	0.544722	-0.26820
$A_{Mp-dipole}$	1.	0.991586	0.977073	0.980147	1.032083	1.042908	0.508400
A_{Mn}^{25}	1.	0.995531	0.986748	1.017259	1.034998	0.911895	0.729953
A_{Mn}^{43}	1.	0.995911	0.985066	1.018644	1.030693	0.907969	0.955653
A_{En}^{25}	1.	1.101871	1.137845	1.019028	1.103693	1.522403	0.970600
A_{En}^{43}	1.	1.101871	1.137338	1.022130	1.098976	1.518870	1.270800
$A_{FA}^{25-dipole}$	1.0000	0.913266	0.995466	1.104324	1.175318	1.391203	0.744317

TABLE II: Fit parameters for $A_N(\xi)$, the LaGrange portion of the new parameterization. Note A_{Mn}^{25}, A_{En}^{25}, and A_{FA}^{25} are constrained to have $\frac{d}{u} = 0$ at $\xi = 1$, and A_{Mn}^{43}, A_{En}^{43}, are constrained to have $\frac{d}{u} = 0.2$.

$Experiment$	M_A (published)	ΔM_A new–old
$Miller - D - ANL_{82,77,73}$	1.00 ± 0.05	-0.035
$Baker - D - BNL_{81}$	1.07 ± 0.06	-0.032
$Kitagaki - D - FNAL_{83}$	$1.05^{+0.12}_{-0.16}$	-0.024
$Kitagaki - D - BNL_{90}$	$1.070^{+0.040}_{-0.045}$	-0.039

TABLE III: M_A (GeV/c^2) values published by neutrino-deuterium experiments[13] and updated corrections ΔM_A when re-extracted with updated "BBBA2007" form factors.

0.4286). The fit utilizing $\frac{d}{u} = 0$ is G^25_{Mn}, and the fit utilizing $\frac{d}{u} = 0.2$ is G^43_{Mn}. The final parameters are given in Table II (or download computer code at *www.pas.rochester.edu/ bodek/FF/*).

The value $A(\xi)$=p_j at ξ_j=1 for G_{En} is set by another duality-motivated constraint. R is defined as the ratio of deep-inelastic longitudinal and transverse structure functions. In the elastic limit, R takes the form

$$R_n\left(x = 1; Q^2\right) = \frac{4M_N^2}{Q^2}\left(\frac{G_{En}^2}{G_{Mn}^2}\right)$$

For inelastic scattering, as $Q^2 \to \infty$, $R_n = R_p$. If we assume quark-hadron duality, the same should be true for the elastic form factors at ξ=1 ($Q^2 \to \infty$) limit

$$G_{En}^2/G_{Mn}^2 = G_{Ep}^2/G_{Mp}^2$$

In order to constrain the fits to G_{En} at high Q^2 we have assumed that the values of $\frac{G_{En}^2}{G_{Mn}^2}$ are the same as the measured $\frac{G_{Ep}^2}{G_{Mp}^2}$ for the three highest Q^2 data points for G_{Ep}, and included these three "*fake*" data points in the G_{En} fits. In addition equation 1 also yields the following constraint at $\xi = 1$:

$$A_{En}^{25,43}(\xi = 1) = P_j = \left(\frac{b}{a}\right) \times \left(\frac{1 + 4\frac{d}{u}}{4 + \frac{d}{u}}\right)^{1/2}$$

As there are two parameter sets $A_{Mn}^{25,43}(\xi)$, we have produced two parameter sets $A_{En}^{25,43}$ as shown in Table II. The new form factors G_{Ep}, G_{Mp}/μ_p, G_{Mn}/μ_n, and G_{En} are plotted in Figure 1 as ratios to the dipole form G_D^V.

As seen in Table II, $A_N(\xi)$ is not needed for G_{Mp} as it is very close to 1.0. For G_{Ep} it yields a correction of 1% at low Q^2 (because it is required to agree with the fits of Arrington and Sick[9] (which include two photon exchange and Coulomb corrections. For G_{En} and G_{Mn} it is used to impose quark-hadron duality asymptotic constraints. Figure 2 shows plots of the data and fits to $\frac{G_{En}}{|G_{Mn}|}$ and $\frac{G_{Ep}}{|G_{Mp}|}$ (for the $\frac{d}{u} = 0$ at $\xi = 1$ case). The long-dashed line is a quark-hadron duality prediction [7].

Using the updated vector form factors, we perform a complete reanalysis of published neutrino quasielastic [13] data on deuterium. (Because of uncertain nuclear corrections, neutrino data on heavier nuclear targets are not used.) We extract new values of M_A (given in Table III), and updated values of $F_A(Q^2)$. The average of the corrected measurements of M_A from Table III is 1.0142 ± 0.0266. This is to be compared to the average value of 1.016 ± 0.016 extracted from pion electroproduction experiments[12] after corrections for hadronic effects. The average of the two average values is 1.0155 ± 0.0136.

For deep-inelastic scattering, the vector and axial parts

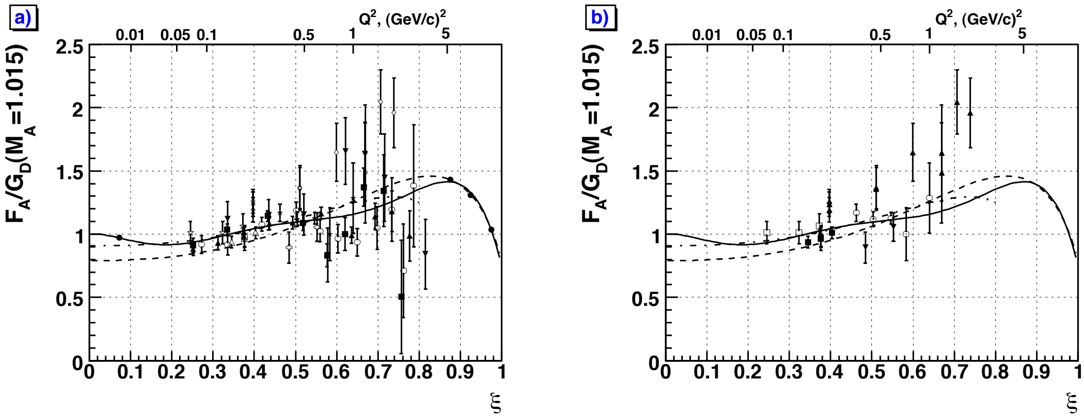

FIG. 3: (a) $F_A(Q^2)$ re-extracted from neutrino-deuterium data divided by $G_D^A(Q^2)$ (with $M_A = 1.015$). (b) $F_A(Q^2)$ from pion electroproduction divided by $G_D^A(Q^2)$ (corrected for for hadronic effects[12]). The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2=V2}$. The dashed-dot line is a prediction from a constituent quark model. The values of ξ and the corresponding values of Q^2 are shown on the bottom and top axis.

of the inelastic structure functions W_2 (or W_1) are equal. Local quark-hadron duality at large Q^2 implies that the axial and vector components of $W_2^{elastic}$ are also equal, which yields:

$$[F_A(Q^2)_{A2=V2}]^2 = \frac{(G_E^V)^2(Q^2) + \tau(G_M^V(Q^2))^2}{(1+\tau)},$$

where $G_E^V(Q^2) = G_{Ep}(Q^2) - G_{En}(Q^2)$ and $G_M^V(Q^2) = G_{Mp}(Q^2) - G_{Mn}(Q^2)$.

We do a duality based fit to the updated values of the axial form factor $F_A(Q^2)$, including pion electroproduction data. Here our fit function is a sum of LaGrange polynomials, A^a_{FA}, multiplied by $G_D^A(Q^2)$ (with $M_A = 1.015$).

$$F_A(Q^2) = A^a_{FA}(\xi) \times G_D^A(Q^2).$$

We impose the constraint $A^a_{FA}(\xi_1 = 0) = p_1 = 1.0$. We also constrain the fit by requiring that $A^a_{FA}(\xi)$ yield $F_A(Q^2) = F_A(Q^2)_{A2=V2}$ (by including additional $''fake''$ data points) for $\xi > 0.9$ ($Q^2 > 7.2 (GeV/c)^2$). Figure 3(a) shows values of $F_A(Q^2)$ extracted from neutrino-deuterium experiments divided by $G_D^A(Q^2)$, with $M_A = 1.015$. Figure 3(b) shows values of $F_A(Q^2)$ extracted from pion electroproduction experiments divided by $G_D^A(Q^2)$, with $M_A = 1.015$. These pion electroproduction values can be directly compared to the neutrino results because they are multiplied by a factor $F_A(Q^2, M_A = 1.014)/F_A(Q^2, M_A = 1.069)$ to correct for $\Delta M_A = 0.055$ originating from hadronic effects[12]. The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2=V2}$. The dashed-dot line is a prediction from a constituent quark model[14].

In summary, Our new parameterizations are useful in modeling neutrino interactions. at low energies. (e.g. for neutrino oscillations experiments). Our predictions for $G_{En}(Q^2)$ and $F_A(Q^2)$ can be tested in future $e-N$ and ν-N experiments.

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