Classification of the Flipped $SU(5)$ Heterotic-String Vacua

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Outline

- Heterotic String Phenomenology
- Free Fermionic Construction
- Flipped $SU(5)$ Models
- Classification
- Future Work

Heterotic String Phenomenology

Free Fermionic Construction

- 4D Theory
- $N = 1$ Supersymmetry
- 3 Generation Standard Model Fermions
- SO(10) GUTs
- Absence of exotic states

Free Fermionic Construction

Properties

- Conformally invariance
- Decoupling left and right moving modes
- D $= 4$ theory

Result

- $C_L = -26 + 11 + D + \frac{D}{2} + \frac{N_{f_L}}{2} = 0$

  $\Longrightarrow$ 18 left-moving real fermions
- $C_R = 0$

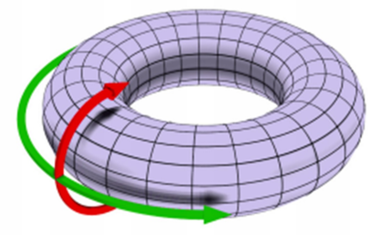
  $\Longrightarrow$ 44 right-moving real fermions

Free Fermionic Construction

- Partition function is used to include all physical states

$$Z = \sum_{\alpha,\beta} C\begin{pmatrix}\alpha \\ \beta\end{pmatrix} Z\left[\alpha,\beta\right]$$

- Taking the one-loop partition function transforms the worldsheet into a torus.



Free Fermionic Construction

$$\alpha = \left\{ \psi^{1,2}, \chi^i, y^i, \omega^i | \overline{y}^i, \overline{\omega}^i, \overline{\psi}^{1,..,5}, \overline{\eta}^{1,2,3}, \overline{\phi}^{1,..,8} \right\}$$

Where $i = 1, ..., 6$

- Left-movers
  - X_L^μ, $\mu = 1, 2$ 2 transverse coordinates
  - ψ_L^μ, $\mu = 1, 2$ The fermionic partners
  - Ω^j, $j = 1, .., 18$ 18 internal real fermions
- Right-movers
  - X_R^μ, $\mu = 1, 2$ 2 transverse coordinates
  - $\overline{\Omega}^j$, $j = 1, .., 44$ 44 internal real fermions

Free Fermionic Construction

- ABK Rules
  - $\sum_i m_i b_i = 0$
  - $N_{ij} \cdot b_i \cdot b_j = \text{mod } 4$
  - $N_i \cdot b_i \cdot b_i = \text{mod } 8$
  - $1 \in \Xi$
  - Even number of fermions
- One-Loop Phases
  - $C\begin{pmatrix} b_i \\ b_j \end{pmatrix} = \pm 1 \text{ or } \pm i$
- GSO Projection
  - $e^{i\pi b_i \cdot F_\alpha} |s\rangle_\alpha = \delta_\alpha C\begin{pmatrix} \alpha \\ b_i \end{pmatrix}^* |s\rangle_\alpha$
- Virasoro Condition
  - $M_L^2 = -\frac{1}{2} + \frac{\alpha_L^2}{8} + \sum \nu_L = -1 + \frac{\alpha_R^2}{8} + \sum \nu_R = M_R^2$

Basis Vectors

- $v_1 = S = \{\psi^\mu, \chi^{1,\dots,6}\}$
- $v_{1+i} = e_i = \{y^i, \omega^i | \overline{y}^i, \overline{\omega}^i\} \qquad i = 1, \dots, 6$
- $v_8 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \overline{y}^{34}, \overline{y}^{56}, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\}$
- $v_9 = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \overline{y}^{12}, \overline{y}^{56}, \overline{\eta}^2, \overline{\psi}^{1,\dots,5}\}$
- $v_{10} = z_1 = \{\overline{\phi}^{1,\dots,4}\}$
- $v_{11} = z_2 = \{\overline{\phi}^{5,\dots,8}\}$
- $v_{12} = \alpha = \{\overline{\eta}^{1,2,3} = \frac{1}{2}, \overline{\psi}^{1,\dots,5} = \frac{1}{2}, \overline{\phi}^{1,\dots,4} = \frac{1}{2}, \overline{\phi}^5 = 1\}$

Gauge Group

$$SU(5) \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3 \times Hidden$$

Enhancements

Depending on the choices of the projection coefficients, extra gauge bosons may arise. These arise with any linear combination of z_1, z_2 and α, which are massless. Such as

$$\mathbf{G} = \left\{ \begin{array}{cccc} z_1, & z_2, & z_1 + z_2, & \alpha, \\ \alpha + z_1, & \alpha + z_2, & \alpha + z_1 + z_2, & x \end{array} \right\}$$

Where

$$x = 2\alpha + z_1 = \{\overline{\psi}^{1,\ldots,5}, \overline{\eta}^{1,2,3}\}$$

Observable Gauge Group Enhancements

x is a sector which can enlarge the observable gauge group. Enhancement takes place when the following conditions are satisfied

Sector Condition
$(z_1 + 2\alpha\|e_i) = (z_1 + 2\alpha\|z_k) = 0$

Enhancement Condition	**Resulting Enhancement**
$(z_1 + 2\alpha\|\alpha) = (z_1 + 2\alpha\|b_2)$	$SU(5)_{obs} \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3 \longrightarrow SU(6) \times SU(2) \times U(1)^2$
$(z_1 + 2\alpha\|\alpha) \neq (z_1 + 2\alpha\|b_2)$	$SU(5)_{obs} \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3 \longrightarrow SO(10) \times U(1)^3$

The pre-stated conditions hold for all $i = 1, ..., 6$.

GGSO Coefficients

$$
(v_i|v_j) \;=\; \begin{array}{c} \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ b_1 \\ b_2 \\ z_1 \\ z_2 \\ \alpha \end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \end{array} \\
\left(\begin{array}{cccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1/2 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & -1/2 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1/2 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1
\end{array}\right)
\end{array}
$$

Where $C\binom{v_i}{v_j} = e^{i\pi(v_i|v_j)}$ $\quad (b_i|b_j) \in \{-\frac{1}{2}, 0, \frac{1}{2}, 1\}$

GGSO Coefficients

	S	e_1	e_2	e_3	e_4	e_5	e_6	b_1	b_2	z_1	z_2	α
S												
e_1			0_{1}	1_{2}	1_{3}	1_{4}	0_{5}	0_{6}		1_{7}		1_{8}
e_2				1_{9}	1_{10}	1_{11}	0_{12}	0_{13}		1_{14}		0_{15}
e_3					0_{16}	1_{17}	0_{18}		0_{19}	1_{20}		0_{21}
e_4						0_{22}	0_{23}		1_{24}	0_{25}		1_{26}
e_5							0_{27}	0_{28}	1_{29}	0_{30}		0_{31}
e_6								1_{32}	1_{33}	0_{34}		1_{35}
b_1									0_{36}		1_{37}	$1/2_{38}$
b_2										0_{39}	1_{40}	$-1/2_{41}$
z_1											0_{42}	0_{43}
z_2												$1/2_{44}$
α												

Chiral Matter

The chiral matter spectrum arises from the twisted sectors. The chiral spinorial representations of the observable $SU(5) \times U(1)$ arise from the sectors

$$\begin{aligned} B^{(1)}_{pqrs} &= S + b_1 + pe_3 + qe_4 + re_5 + se_6, \\ &= \{\psi^\mu, \chi^{12}, (1-p)y^3\overline{y}^3, p\omega^3\overline{\omega}^3, (1-q)y^4\overline{y}^4, q\omega^4\overline{\omega}^4, \\ &\quad (1-r)y^5\overline{y}^5, r\omega^5\overline{\omega}^5, (1-s)y^6\overline{y}^6, s\omega^6\overline{\omega}^6, \overline{\eta}^1, \overline{\psi}^{1,\dots,5}\}, \\ B^{(2)}_{pqrs} &= S + b_2 + pe_1 + qe_2 + re_5 + se_6, \\ B^{(3)}_{pqrs} &= S + b_3 + pe_1 + qe_2 + re_3 + se_4, \end{aligned}$$

Where $p, q, r, s = 0, 1$ and $b_3 = b_1 + b_2 + 2\alpha + z_1$.

The states in the sector $B^{(1)}_{pqrs}$ can be projected out of the spectrum by the GGSO projection of the vectors e_1, e_2, z_1 and z_2. Similarly for all sectors, we can define a projector P such that the states survive when $P = 1$ and are projected out when $P = 0$:

$$P^{(1)}_{pqrs} = \frac{1}{16}\left(1 - C\binom{e_1}{B^{(1)}_{pqrs}}\right)\cdot\left(1 - C\binom{e_2}{B^{(1)}_{pqrs}}\right)\cdot\left(1 - C\binom{z_1}{B^{(1)}_{pqrs}}\right)\cdot\left(1 - C\binom{z_2}{B^{(1)}_{pqrs}}\right)$$

$$P^{(2)}_{pqrs} = \frac{1}{16}\left(1 - C\binom{e_3}{B^{(2)}_{pqrs}}\right)\cdot\left(1 - C\binom{e_4}{B^{(2)}_{pqrs}}\right)\cdot\left(1 - C\binom{z_1}{B^{(2)}_{pqrs}}\right)\cdot\left(1 - C\binom{z_2}{B^{(2)}_{pqrs}}\right)$$

$$P^{(3)}_{pqrs} = \frac{1}{16}\left(1 - C\binom{e_5}{B^{(3)}_{pqrs}}\right)\cdot\left(1 - C\binom{e_6}{B^{(3)}_{pqrs}}\right)\cdot\left(1 - C\binom{z_1}{B^{(3)}_{pqrs}}\right)\cdot\left(1 - C\binom{z_2}{B^{(3)}_{pqrs}}\right)$$

Projectors

These projectors can be expressed as a system of linear equations with p, q, r and s as unknowns. The solutions of a specific system of equations yield the different combinations of p, q, r and s for which sectors survive the GSO projections.

$$\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_1|b_1) \\ (e_2|b_1) \\ (z_1|b_1) \\ (z_2|b_1) \end{pmatrix},$$

$$\begin{pmatrix} (e_3|e_1) & (e_3|e_2) & (e_3|e_5) & (e_3|e_6) \\ (e_4|e_1) & (e_4|e_2) & (e_4|e_5) & (e_4|e_6) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_3|b_2) \\ (e_4|b_2) \\ (z_1|b_2) \\ (z_2|b_2) \end{pmatrix},$$

$$\begin{pmatrix} (e_5|e_1) & (e_5|e_2) & (e_5|e_3) & (e_5|e_4) \\ (e_6|e_1) & (e_6|e_2) & (e_6|e_3) & (e_6|e_4) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_3) & (z_1|e_4) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_3) & (z_2|e_4) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_5|b_3) \\ (e_6|b_3) \\ (z_1|b_3) \\ (z_2|b_3) \end{pmatrix}.$$

These 48 sectors give rise to 16 and $\overline{16}$ representations of $SO(10)$ decomposed under $SU(5) \times U(1)$

$$\mathbf{16} = (\bar{\mathbf{5}}, -\frac{3}{2}) + (\mathbf{10}, +\frac{1}{2}) + (\mathbf{1}, +\frac{5}{2}),$$
$$\overline{\mathbf{16}} = (\mathbf{5}, +\frac{3}{2}) + (\overline{\mathbf{10}}, -\frac{1}{2}) + (\mathbf{1}, -\frac{5}{2}).$$

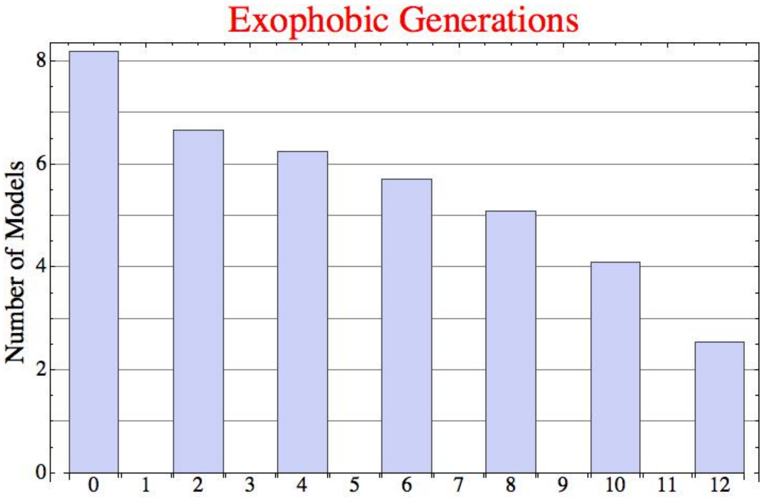
We can decompose the flipped $SU(5)$ representation under $SU(3) \times SU(2) \times U(1)$ as

$$(\bar{\mathbf{5}}, -\frac{3}{2}) = (\bar{3}, 1, -\frac{2}{3})_{u^c} + (1, 2, -\frac{1}{2})_L,$$
$$(\mathbf{10}, +\frac{1}{2}) = (3, 2, +\frac{1}{6})_Q + (\bar{3}, 1, +\frac{1}{3})_{d^c} + (1, 1, 0)_{\nu^c},$$
$$(\mathbf{1}, +\frac{5}{2}) = (1, 1, +1)_{e^c},$$

Classification

- There are 44 independent cooefficients which corresponds to $2^{44} \approx 10^{13}$ different vacua.
- Using computer code we perform a statistical sampling in this space of models and extract 10^{12} distinct configurations with the flipped SU(5) gauge group.

Classification



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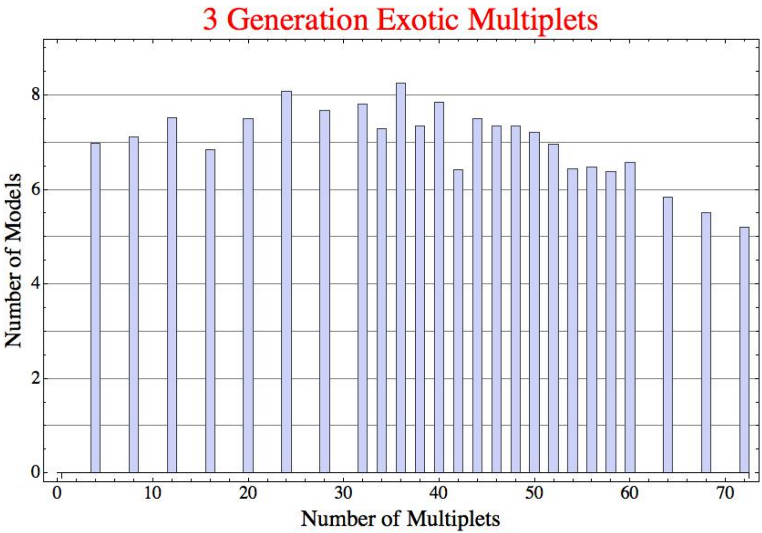

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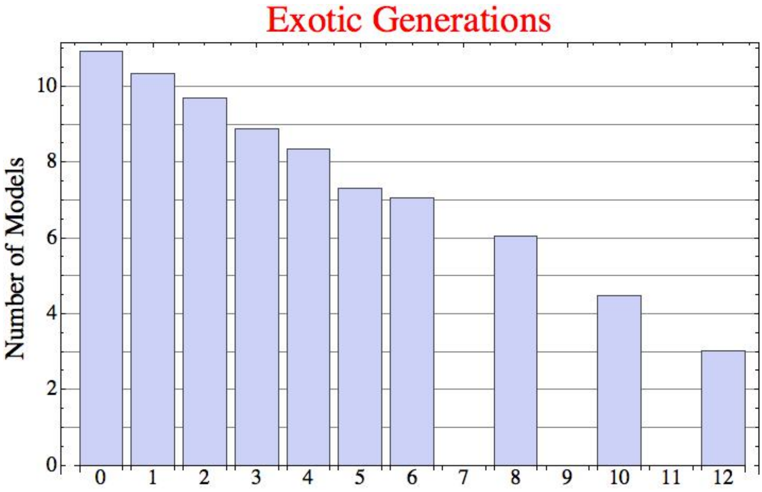

Classification

	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	100000000000	1	1.76×10^{13}
(1)	+ No Enhancements	76226929871	7.62×10^{-1}	1.34×10^{13}
(2)	+ Anomaly Free Flipped $SU(5)$	13954418231	1.40×10^{-1}	2.45×10^{12}
(3)	+ 3 Generations	73804532	7.38×10^{-4}	1.30×10^{10}
(4a)	+ SM Light Higgs	70639603	7.06×10^{-4}	1.24×10^{10}
(4b)	+ Flipped $SU(5)$ Heavy Higgs	4647013	4.65×10^{-5}	8.18×10^{8}
(5)	+ SM Light Higgs + & Heavy Higgs	4362491	4.36×10^{-5}	7.67×10^{8}
(6a)	+ Minimal Flipped $SU(5)$ Heavy Higgs	4231039	4.23×10^{-5}	7.44×10^{8}
(6b)	+ Minimal SM Light Higgs	2533321	2.53×10^{-5}	4.46×10^{8}
(7)	+ Minimal Flipped $SU(5)$ Heavy Higgs + & Minimal SM Light Higgs	2463689	2.46×10^{-5}	4.33×10^{8}
(8)	+ Minimal Exotic States	121868	1.22×10^{-6}	2.14×10^{7}

Conclusion

Future work

- Extentions on Pati-Salam models
- $SU(6) \times SU(2)$
- $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$
- Other Gauge Groups
- Compute the Superpotentials

THANK YOU